

I. *An Account of the Cause of the late remarkable Appearance of the Planet Venus, seen this Summer, for many Days together, in the Day time.* By Edm. Halley, R. S. Secr.

IT may justly be reckoned one of the principal Uses of the Mathematical Sciences, that they are in many Cases able to prevent the Superstition of the unskilful Vulgar; and by shewing the genuine Causes of rare Appearances, to deliver them from the vain apprehensions they are apt to entertain of what they call *Prodigies*; which sometimes, by the Artifices of designing Men, have been made use of to very evil purposes.

Of this kind was the late Appearance of *Venus* in the Day time, generally taken notice of about *London* and elsewhere; and by some reckoned to be *Prodigious*. This put me upon the enquiry, how it came to pass that at that time the *Planet* should be so plainly seen by Day, whereas she rarely shews her self so, unless to those who know exactly where to look for her. To resolve this, the following Problem arose, *viz.* To find the Situation of the *Planet* in respect of the *Earth*, when the *Area* of the illuminated part of her Disk is a *Maximum*.

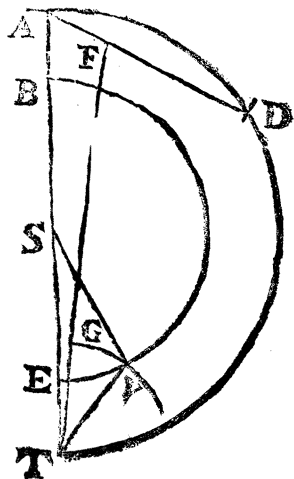
To investigate this *Maximum*, I found it requisite to assume the following *Lemmata*. I. That the visible *Areas* of the Disk of the same *Planet*, at differing Distances, are always reciprocally as the Squares of those Distances; which is evident from the first Principles of *Opticks*. II. That the *Area* of the whole Disk of the *Planet* is to the *Area* of the illuminated Part thereof, as the Diameter of a Circle to the Versed-Sine of the exterior Angle at the *Planet*, in the Triangle at whose Angles are the *Sun*, *Earth*, and *Planet*. III. That in all plain Triangles, four times the Rectangle of the Sides containing any Angle, is to the excess of the Square of the Sum of the Sides above the Square of the Base, as the Diameter is to the Versed-Sine of the

the Complement of the contained Angle to a Semicircle, which I call the *exteriour Angle*: This is a new *Theorem* of good use in *Trigonometry*, and easily proved from the 12th and 13th of the II. *Elem. Euclid*.

This premised, putting m for the Distance of the *Sun* and *Earth*, and n for that of the *Sun* and *Venus*, and x for the Distance of the *Earth* and *Venus*, or the third Side of the Triangle which we seek; by the third *Lemma*, $4nx$, will be to the excess of the Square of $n+x$ above the Square of m , as the *Area* of the whole Disk of *Venus* to the *Area* of the part illuminated; and by the first *Lemma*, the *Area's* of her whole Disk are at all times as the Squares of x reciprocally; whence the Quantity $\frac{nn + 2nx + xx - mm}{4nx^3}$ will in all Cases be proportional to the *Area* of the illuminated part.

Now that this should be a *Maximum*, it is required that the Fluxion thereof be equal to 0, or that the *Negative* parts thereof be equal to the *Affirmative*, that is, that $2nx + 2xx \times 4nx^3 = 12nx^2x \times nn + 2nx + xx - mm$; and dividing all by $4nx^2x$, the Equation becomes $2nx + 2xx = 3nn + 6nx + 3xx - 3mm$. Consequently $3nn + 4nx + xx = 3mm$, and therefore $x = \sqrt{3mm + nn} - 2n$.

From hence a ready and not inelegant Geometrical Construction (if I may be allowed to say so) becomes obvious; for with the Center S and Radius $ST = m$, describe the Semicircle $TD A$; and with the same Center and Radius $SE = n$, the Semicircle $EV B$; which two Semicircles shall represent the Orbs of the *Earth* and *Venus*. Make the chord AD equal to the Radius ST , and from D towards A , lay off $DF = SE$; draw TF , and thereon place $FG = BE = 2n$, and with the Center T and Radius



TG describe the arch *GV*, cutting the Semicircle *BVE* in *V*; and draw the lines *SV*, *TV*: I say the Triangle *STV* is Similar to that at whose Angles are the *Sun*, *Earth* and *Venus*, at the time when the *Area* of the enlightened part of that Planet's Disk, as seen from the *Earth*, is greatest. How this Geometrical Effect follows from the Equation is too evident to need repetition.

In consequence of this Solution, I find this *Maximum* always to happen, when the Planet is about forty Degrees distant from the *Sun*; and the times thereof, about the middle between her greatest Elongations on both sides from him, and her retrograde Conjunctions with him; when little more than a quarter of her visible Disk is luminous, and resembling the Moon of about five Days old; and notwithstanding that her Diameter is at that time but 50 Seconds, yet she shines with so strong a Beam, as to surpass the united light of all the fixt Stars that appear with her, and casts a very strong Shade on the Horizontal plain whereon they all shine: an irrefragable Argument to prove that the Disks of the fixt Stars are unconceivably small, and next to nothing; since shining with a native Light, so many of them do not equal the reflex Light of one quarter of a Disk of less than a Minute Diameter.

In this situation *Venus* was found in *July* last, on the tenth Day, about which time, when the *Sun* grew low, she was very plainly seen in the Day time, for many Days together; as she might have been in the Mornings, about the latter end of *September*. But this, arising from the Causes we have now shewn, is nothing uncommon; for every eighth Year it returns again, so that the Planet may be seen on the same Day of the Month and Hour, very nearly in the same place; as all acquainted with the Heavenly Motions must know.

Lastly, it may not be amiss to note that the Equation $x = \sqrt{3mm + nn} - 2n$ has a Limit; for if n be equal to $\frac{1}{4}m$, the point *V* will fall on *B*; and the whole Disk of a Planet at that distance from the Sun would be the *Maximum*, viz. when in its superior Conjunction with the *Sun*. And the like if n were less than $\frac{1}{4}m$; the Arch *GV* in such Case not intersecting the Semicircle *BE*.